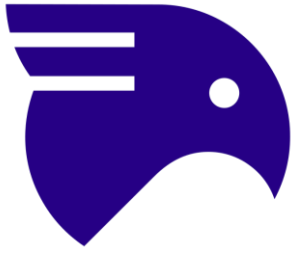


Proofs are Programs

Summer 2026 @ Ruhr Uni Bochum




Lecturers: Cătălin Hriţcu and Clara Schneidewind

TAs: Yonghyun Kim and Yan Farba

Max Planck Institute for Security and Privacy (MPI-SP)

This course in a nutshell

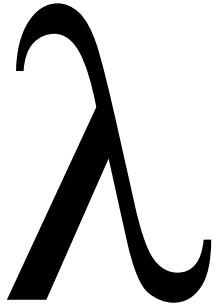
- Logic and proofs
- Verifying functional programs
- Formalizing simple imperative language
- Secure information flow
- Using the Rocq proof assistant
  - gentle introduction to both functional programming and formal proofs (both in Rocq)

The Rocq proof assistant

["Le Coq mécanisé" picture by Lilia Anisimova]

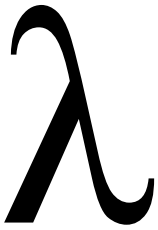
- Rocq developed at Inria in France since 1983 (in OCaml)
  - Formerly known as Coq (which in French means rooster)
- **Rocq helps you build formal proofs interactively**
- **Proving in Rocq is like programming**
  - gamified, addictive, and lots of fun
  - if you like programming, you will also like Rocq proofs
- **This helps you deeply understand proofs**
- **In fact, formal Rocq proofs are just purely functional programs**
  - **Curry-Howard correspondence (two extra chapters)**: deep connection between logic and functional programming

Functional programming



- **Write computations as recursive functions**
  - **using recursion, immutable datatypes, and pattern matching**
  - **limit side-effects**, such as **mutating** stateful data structures
- **Rocq is purely functional = zero side-effects**
  - all computations are mathematical functions (in particular terminating)
- **Functional programming languages** like OCaml, Haskell, ...
  - not completely pure, but still try to reduce and control side-effects
  - they still make it as easy as possible to write functional code
- **BSc course previous semester using OCaml** (again next semester)
  - Not a prerequisite for this course though

Functional Programming in practice



- **Functional languages have some practical success**
  - **Meta** (OCaml, Haskell, Rust), **Microsoft** (OCaml, F#, F*, and Rust), **X** (Scala), **Mozilla** (Rust), **Google** (Rust), **Amazon** (Rust), **Financial industry**, **Blockchains and smart contracts** (Clara's group does research on this), ...
- **Not yet fully mainstream, but ...**

  - **Many cool ideas already adopted by mainstream languages**:
    - Lambdas, Generics in Java/C#, Rust's type system, datatypes, pattern matching (most admired language on Stack Overflow for the last 11 years!)
  - **Functional programmers often earn more** (Stack Overflow developer survey)
  - **Functional programs are concise, elegant, beautiful**
    - This makes understanding and reasoning about programs much easier, both informally and formally
- **For this course this last point is very important**
  - formal verification by proving theorems about purely functional programs in Rocq

ROCQ proof that append is associative

```
Fixpoint app {a:Type} (l1 l2 : list a) : list a :=
  match l1 with
  | [] => l2
  | hd :: tl => hd :: (app tl l2)
  end.


Theorem app_assoc : forall (a:Type) (l1 l2 l3 : list a),
  app (app l1 l2) l3 = app l1 (app l2 l3).
Proof.
  intros a l1 l2 l3. induction l1 as [| hd tl IH].
  - (* l1 = [] *) simpl. reflexivity.
  - (* l1 = hd :: tl *) simpl. rewrite -> IH. reflexivity.
Qed.
```

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Qed.
```



```
1 goal (ID 25)

- a : Type
- hd : a
- tl, l2, l3 : list a
- IH : app (app tl l2) l3 = app tl (app l2 l3)
  ============================
  hd :: app (app tl l2) l3 = hd :: app tl (app l2 l3)
```

- **Define inductive datatypes**
  - bools, numbers, lists, trees, abstract syntax trees
- **Define pure functions on these datatypes**
  - pattern matching & structural recursion (terminating)

```
Fixpoint app {a:Type} (l1 l2 : list a) : list a :=
  match l1 with
  | [] => l2
  | hd :: tl => hd :: (app tl l2)
  end.
```

- **Prove properties of these functions by induction**

```
Theorem app_assoc : forall (a:Type) (l1 l2 l3 : list a),
  app (app l1 l2) l3 = app l1 (app l2 l3).
Proof.
  intros a l1 l2 l3. induction l1 as [| hd tl IH].
  - (* l1 = [] *) simpl. reflexivity.
  - (* l1 = hd :: tl *) simpl. rewrite -> IH. reflexivity.
Qed.
```

Formalizing simple imperative language

- **Abstract syntax tree (inductive datatype)**

```
Inductive com : Type :=
  | CSkip
  | CAsgn (x : string) (a : aexp)
  | CSeq (c1 c2 : com)
  | CIf (b : bexp) (c1 c2 : com)
  | CWhile (b : bexp) (c : com).

Notation "'if' b 'then' c1 'else' c2 'end'"
  := (CIf b c1 c2) ...
```

- **Operational semantics (inductive relation)**

$$\frac{\texttt{beval st b = true} \quad \texttt{st =[} c_1 \texttt{]=> st'}}{\texttt{st =[if b then } c_1 \texttt{ else } c_2 \texttt{ end]=> st'}}\ \text{(E\_IfTrue)}$$

$$\frac{\texttt{beval st b = false} \quad \texttt{st =[} c_2 \texttt{]=> st'}}{\texttt{st =[if b then } c_1 \texttt{ else } c_2 \texttt{ end]=> st'}}\ \text{(E\_IfFalse)}$$

- **Proving properties of some/all programs**
  - **e.g. in our simple language evaluation is deterministic:**

```
Theorem ceval_deterministic: forall c st st1 st2,
  st =[ c ]=> st1 ->
  st =[ c ]=> st2 ->
  st1 = st2.
```

  - **by induction** (often on derivation trees of inductive relation)

Inductive relation

$$\frac{}{st =[\ skip\ ]\Rightarrow st}\ (E\_Skip)$$

$$\frac{aeval\ st\ a = n}{st =[\ x := a\ ]\Rightarrow (x\ !\to n\ ;\ st)}\ (E\_Asgn)$$

$$\frac{st =[\ c_1\ ]\Rightarrow st' \quad st' =[\ c_2\ ]\Rightarrow st''}{st =[\ c_1;c_2\ ]\Rightarrow st''}\ (E\_Seq)$$

$$\frac{beval\ st\ b = true \quad st =[\ c_1\ ]\Rightarrow st'}{st =[\ if\ b\ then\ c_1\ else\ c_2\ end\ ]\Rightarrow st'}\ (E\_IfTrue)$$

$$\frac{beval\ st\ b = false \quad st =[\ c_2\ ]\Rightarrow st'}{st =[\ if\ b\ then\ c_1\ else\ c_2\ end\ ]\Rightarrow st'}\ (E\_IfFalse)$$

$$\frac{beval\ st\ b = false}{st =[\ while\ b\ do\ c\ end\ ]\Rightarrow st}\ (E\_WhileFalse)$$

$$\frac{beval\ st\ b = true \quad st =[\ c\ ]\Rightarrow st' \quad st' =[\ while\ b\ do\ c\ end\ ]\Rightarrow st''}{st =[\ while\ b\ do\ c\ end\ ]\Rightarrow st''}\ (E\_WhileTrue)$$

Formalized in ROCQ

```
Inductive ceval : com -> state -> state -> Prop :=
  | E_Skip : forall st,
      st =[ skip ]=> st
  | E_Asgn  : forall st a n x,
      aeval st a = n ->
      st =[ x := a ]=> (x !-> n ; st)
  | E_Seq : forall c1 c2 st st' st'',
      st  =[ c1 ]=> st'  ->
      st' =[ c2 ]=> st'' ->
      st  =[ c1 ; c2 ]=> st''
  | E_IfTrue : forall st st' b c1 c2,
      beval st b = true ->
      st =[ c1 ]=> st' ->
      st =[ if b then c1 else c2 end]=> st'
  | E_IfFalse : forall st st' b c1 c2,
      beval st b = false ->
      st =[ c2 ]=> st' ->
      st =[ if b then c1 else c2 end]=> st'
  | E_WhileFalse : forall b st c,
      beval st b = false ->
      st =[ while b do c end ]=> st
  | E_WhileTrue : forall st st' st'' b c,
      beval st b = true ->
      st  =[ c ]=> st' ->
      st' =[ while b do c end ]=> st'' ->
      st  =[ while b do c end ]=> st''

where "st =[ c ]=> st'" := (ceval c st st').
```

Why formalize programming languages?

CompCert C compiler

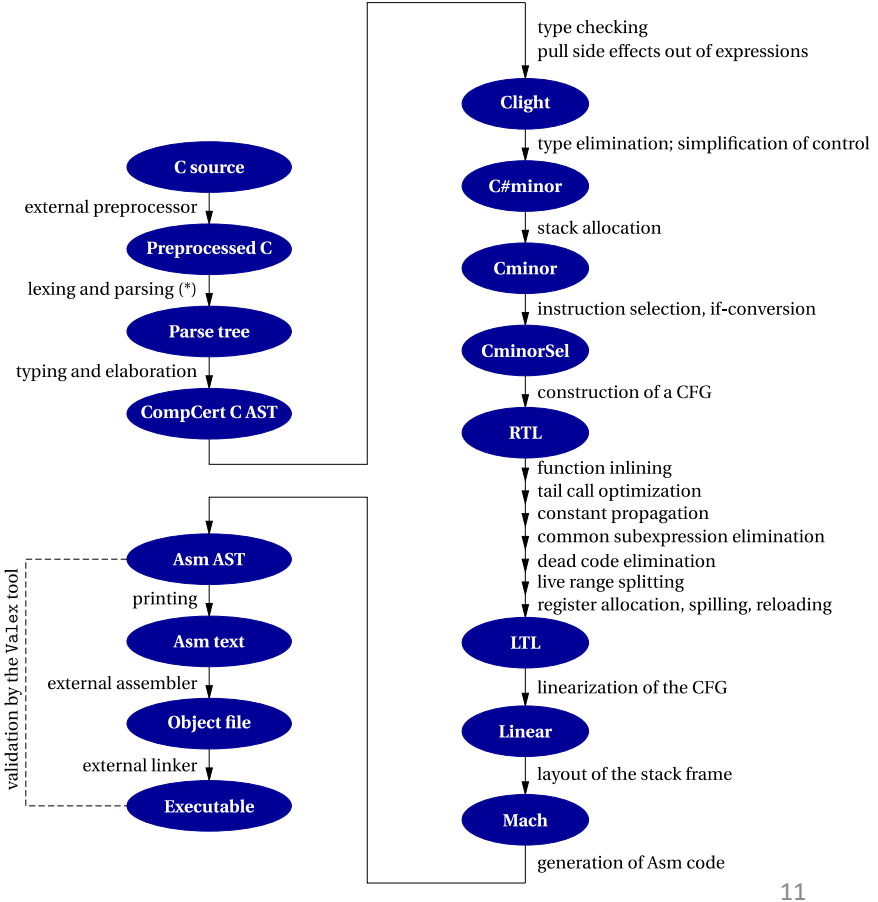
verified in Rocq to compile correctly

- each language given a semantics
- transformations and optimizations
  - implemented as pure functions
  - proved to preserve semantics
- used in safety-critical systems

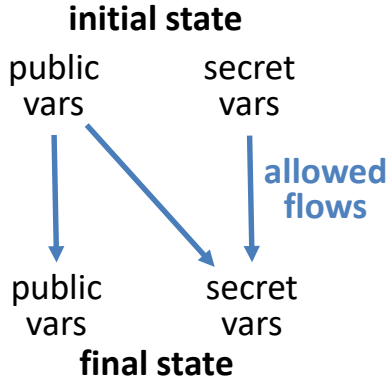
Cătălin, Yonghyun, Yan, et al building secure compilers and verifying them in Rocq

- including a secure variant of CompCert
- side-channel defenses (more later)

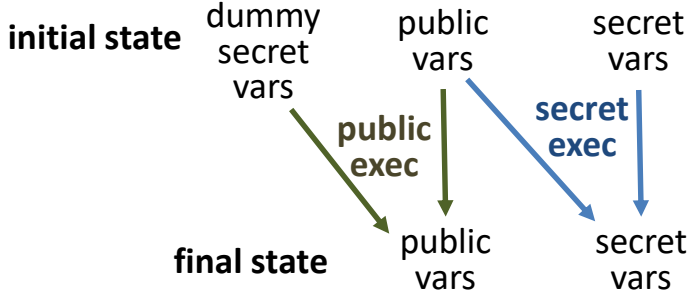
Clara et al built secure compiler from BitMLx smart contracts to cross-chain contracts and protocols



Secure information flow



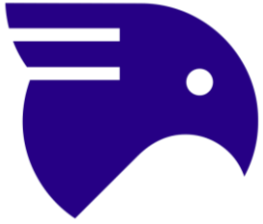
- **What does it mean that a program doesn't leak secrets?**
- **Noninterference** (for simple imperative programs):
  - secrets don't flow from secret variables to public variables
  - Formally: executing the program twice with different initial values for the secret variables produces two final states whose public variables are still equal
- **Can be enforced statically by simple type system**
  - Prevent explicit flows: public := 2*secret + 1
  - Prevent implicit flows, via the control flow:
    if secret=0 **then** public := 0 **else** public := 1
- **Can also be enforced dynamically**
  - e.g. Secure Multi-Execution
  - secure and precise,
    but less efficient and less correct



More realistic leakage models for information flow

- **Cryptographic constant time** (i.e. secret independent timing)
  - **widely-used programming discipline for writing cryptographic code** without leaking secrets via obvious cache **side channels**:
    - no secret-dependent branches and no secret-dependent memory accesses
  - the obtained guarantees formalized as a **variant of noninterference**
  - can also be checked by a **simple type system**
- **Speculative security** (Cătălin, Yonghyun, Yan, et al and Jana et al.)
  - Spectre: constant time code can still leak because of speculative execution
  - **Speculative constant time**
    - **variant of noninterference** that prevents leaks in speculative executions
  - **Speculative load hardening** transformation enforcing this security property

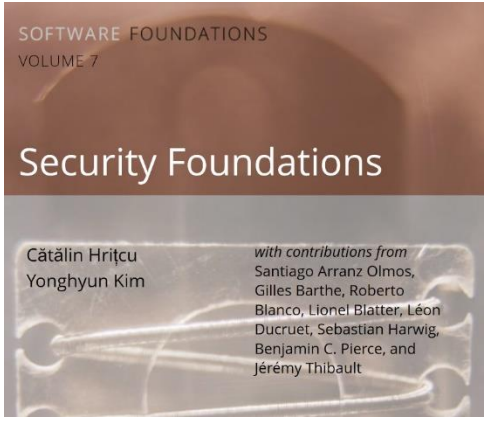
In this course you will learn ...



- **to write purely functional programs in Rocq**
  - natural numbers, lists, trees, program syntax
- **to verify these programs by proving theorems about them**
  - case analysis, induction, inversion, proof automation, ...
- **to formalize a simple imperative programming language**
  - syntax and operational semantics (evaluation derivations)
- **to make programs secure with information-flow control**
  - enforcing noninterference statically or dynamically
  - even against side-channels and speculative executions attacks (optional for BSc students)

This course is very hands on

- **Rocq is your personal proof assistant**
  - Taking gamification to the next level!
- **Course based on two textbook volumes**
  - **lots of exercises in Rocq**,
    from very simple to more challenging ones
  - our volume versions available at:
    https://mpi-sp-pap-2026.github.io/book-lf
    https://mpi-sp-pap-2026.github.io/book-secf
- **You will understand proving**
  - It's not hard, it's just programming!
  - If you like programming,
    you will like formal proofs too!
- **Advice: ask questions, interact**



Lecture logistics

- 14 lectures: roughly first 1/2 Clara, second 1/2 Cătălin
  - exceptions: first lecture mostly Cătălin, Imp lecture Clara, ...
- Pentecost Vacation 25-29 May, so no lecture, no tutorials
- We hope for a mostly in-person course
  - **So please attend physically whenever possible!**
  - When you **really cannot** attend physically
    you can use Zoom or watch the recording (see Moodle)
- **Join on Moodle for all materials**
  - **If external to RUB please create account with any email address**
- **Advice: ask questions, interact during the lecture**

Exercises

- **Solving exercises strongly recommended**
  - you will learn the most by writing programs and proofs in Rocq
  - very strong correlation between exercise scores and exam scores
  - **highly recommended even if you're not taking this for credit**
- **Exercises count for up to 10% of bonus points**
  - not required to do the optional exercises; they don't count for grade
- **New exercise sheet will be released on Moodle after all courses**
  - there will be around 13 exercise sheets in total
- You have to **turn in your solution on Moodle before next course**
  - **up to Wednesday at 11:59 AM**
- **Exercises are individual, please don't share solutions in any way!**
- **Using generative AI to solve homework exercises is not allowed!**

Tutorials: Q&A about the exercise sheets

- **TAs:** Yonghyun Kim and Yan Farba
- **Tuesdays at 10:15-11:45, on the 5th floor of this building**
- You can come and **ask existing questions**
  - Can also ask about old assignments, but solutions anyway on Moodle
- You can also **work on your own during tutorials**
  - **and ask questions as they arise**
- If you manage to solve an exercise sheet and don't have any questions, then no problem, **you are not forced to come**
- **Zoom participation in Q&A sessions possible** (same Zoom room)
  - **if you cannot make it in person**, but in-person participants get priority

Exams

- **Midterm exam** (optional)
  - practice for the final exam
  - also written, on paper
  - duration: 60 minutes
  - bonus points: up to 10%
  - date: Tue, 2 June
    - time: 10:15-11:15

- **Final exam**
  - written, on paper
    - so we will also teach you how to write down proofs informally
  - duration: 120 minutes
  - 100% of the grade
  - date: 7 August
  - re-exam: 29 September

Credit and grade

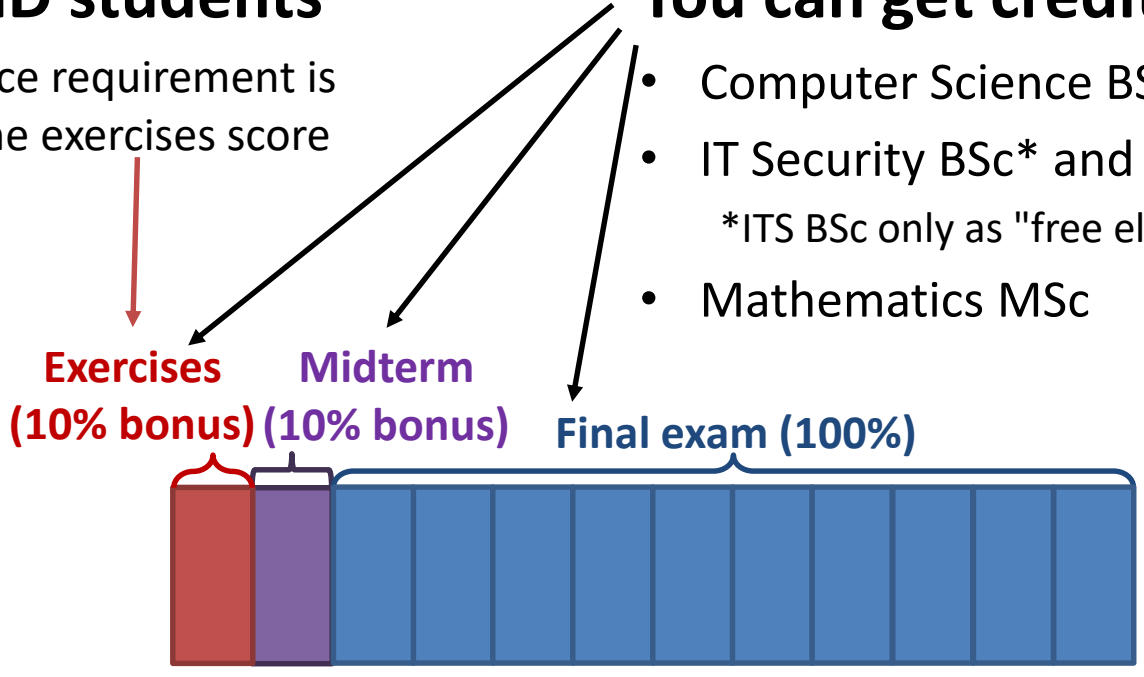
CASA PhD students

- attendance requirement is 50% of the exercises score

You can get credit if you study

- Computer Science BSc and MSc
- IT Security BSc* and MSc

  *ITS BSc only as "free elective" (sorry!)
- Mathematics MSc



Adding up <u>everything</u>, you need 49.01% to pass and get credit,

and you need at least 94.01% to get highest grade

BACKUP SLIDES

Logics and proofs

Q: How do we know something is true?

A: We prove it

Q: How do we know that we have a **proof**?

A: We need to know what it means for something to be a proof.
First cut: A proof is a "logical" sequence of arguments,
starting from some initial assumptions

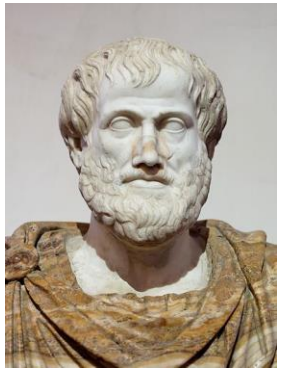
Q: How do we agree on what is a **valid** sequence of arguments?
Can any sequence be a proof? E.g.

All humans are mortal

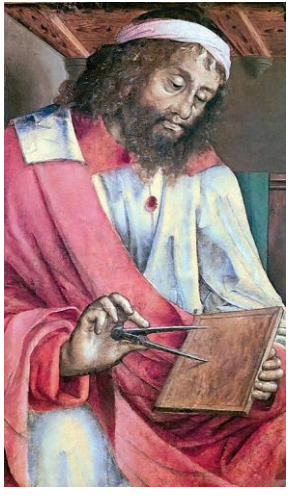
All Greeks are human

Therefore I am a Greek!

A: No, no, no! We need to think harder about **valid** ways of reasoning...



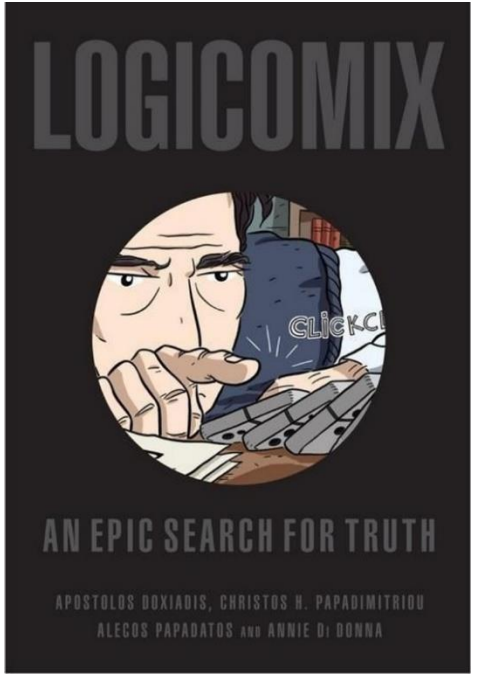
Aristotle
384 – 322 BC



Euclid
~300 BC

The story of logics is quite fascinating

- **David Hilbert** (1862 – 1943)
- **Gottlob Frege** (1848-1925)
  - first-order logic (∀x. P x /\ R x x)
  - derive the laws of arithmetic from first principles (Hilbert's 2nd problem)
- **Bertrand Russell** (1872 - 1970)
  - paradox, inconsistency in Frege's logical system, tried to fix it
  - Principia Mathematica
- **Kurt Gödel** (1906 - 1978)
  - incompleteness theorems



- **Gerhard Gentzen** (1909-1945) -- consistency of arithmetic
- **Alonzo Church** (1903 - 1995) -- lambda calculus, simple theory of types
- **Alan Turing** (1912 - 1954) -- undecidability of arithmetic, halting problem
- ...

Logics and computer science

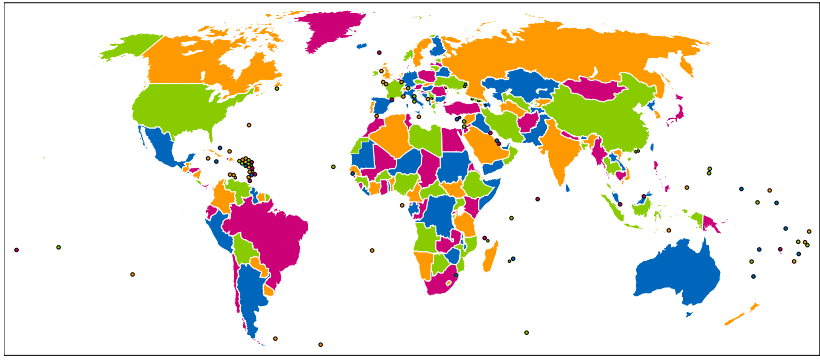
- Logics is a foundation of **mathematics** and **computer science**
  - **precise proofs** with respect to **valid inference steps/rules**
- **Logics and CS greatly influenced each other**, e.g.:
  - **automated theorem provers** (e.g., SAT and SMT solvers)
  - **model checkers**
  - **proof assistants**: Rocq, Isabelle, HOL family, F*, ACL2, etc.
    - interactively constructed, machine-checked proofs
    - addictive, gamification of proofs

Logics and proofs

- Foundation of **mathematics** and **computer science**
  - **formal proofs** with respect to **valid inference steps/rules**
- **This course: typed constructive higher-order logic**
  - **higher-order**
    - can quantify not only over individuals (∀x:nat. P x)
    - but also over propositions (∀P:Prop. P),
      predicates (∀Q:nat->Prop. Q x), relations (∀R:nat->nat->Prop. R x y),
      functions (∀f:nat->bool, ∀g:(nat->bool)->bool), ...
  - **constructive**, aka **intuitionistic logic**:
    - a proposition is valid if one can construct a proof
    - philosophically rejects excluded middle (P ∨ ¬P, classical logic)
  - **typed** (dependently typed)

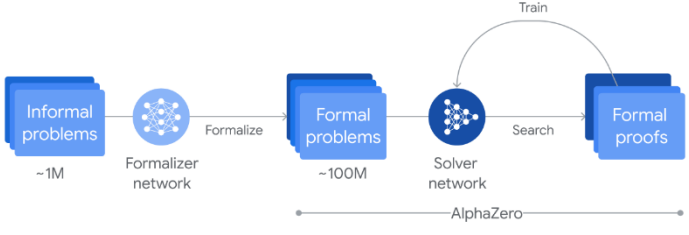
Machine-checked proofs of math theorems

- **The 4-color theorem**
  - proved in 1976, simplified in 1996
  - proofs rely on a computer
  - infeasible for a human to check
  - initially not accepted by all mathematicians
  - mechanized in Rocq by Georges Gonthier in 2005



[World map from Wikipedia]

- **Construction of perfectoid spaces** (Peter Scholze, Lean community, 2020)
  - Mathlib math library for Lean has over 100,000 theorems, over 1,000,000 lines of code, and an active community including many mathematicians (e.g. Terence Tao)
- **AlphaProof: Google DeepMind AI achieves silver medalist level in IMO (2024)**
  - Key ingredient: the Lean proof assistant checks each step of the AI



Verifying realistic imperative programs

- **the seL4 OS microkernel** (Isabelle/HOL), **the CertiKOS hypervisor** (Rocq)
- **EverCrypt cryptographic library** (F*) -- shipping in Firefox and the Linux kernel
- **EverParse3D verified binary parsers** (F*) -- shipping in the Windows kernel
  - Cătălin, Cezar, et al designing and using F* proof assistant, which combines features of Rocq (dependent types) with Hoare Logic verification (Dijkstra monads)
- **Libjade high-assurance post-quantum crypto library** (EasyCrypt, MPI-SP et al)
  - EasyCrypt proof assistant using Probabilistic Relational Hoare Logic (MPI-SP et al)
- **Verification of smart contracts** (Rocq, F*, ...)
  - Clara, Jana, and their groups at MPI-SP working on this hot topic